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THE USE OF A TWO-DEGREES-OF-FREEDOM GYROSCOPE AS A SATELLITE YAW SENSOR

SUMMARY

The ability of a two-degrees-of-freedom gyroscope to indicate yaw attitude in an orbiting earth-pointed vehicle is evaluated. The gyro is first studied as a separate orbiting unit under the influence of gimbal angle damping and torquing, and it is shown that the gyro indicates yaw position. The gyro is then inserted in the yaw attitude control loop of a vehicle in circular orbit. The stability characteristics are studied, and analog time histories illustrate the system dynamics and yaw sensing characteristics of the gyro. The ability of the two-degrees-of-freedom gyro to indicate large yaw-angle offsets and to supply stabilizing information to the vehicle for these angles is described. Finally, a comparison is made between the performance of a twc-degrees-of-freedom gyroscope and a roll-rate gyroscope as used in the vehicle yaw control loop. The two-degrees-of-freedom gyroscope shows considerable improvement over the roll-rate gyroscope.

INTRODUCTION

Active control to keep an axis of a satellite vehicle pointed along the vertical direction toward the earth poses no fundamental problem. Horizon sensors afford roll and pitch error signals that are adequate, at least in principle, to control a vehicle's vertical alinement. The situation, however, is different for yaw. Here it may be desired to keep an axis of the vehicle heading in the orbital plane more or less along the velocity vector.

Some methods for controlling or determining this yaw heading angle proposed and analyzed in the past have depended for their success on the orbital angular momentum or angular velocity coupling inherent in a vertically oriented satellite. The angular momentum of the vehicle so couples its yaw and roll modes that motion about one axis is a function of motion about all axes. If zero roll rate is maintained by some means, then a roll-rate gyro reads the component of the orbital angular velocity in proportion to the magnitude of the yaw angle. This method of determining vehicle yaw attitude may often be sufficient to satisfy a particular requirement, but it is not entirely satisfactory for all systems.

The present paper considers another method of determining yaw attitude for use in an earth pointed satellite. In place of a rate gyro mounted with its sensitive axis along the vehicle's roll axis, a two-degrees-of-freedom gyroscope is considered with its spin axis parallel to the orbital angular momentum vector. Success of the method depends on using a vertical reference to maintain the gyro's inner gimbal in the local horizontal plane. Then the gyro acts like a gyrocompass, alining its spin vector in a stable position with the orbital angular momentum vector. The gyro is statically stable and self-alining. The yaw angle measured by the outer gimbal angle can also be used by a control system to maintain a desired vehicle yaw angle.

The report describes the two-degrees-of-freedom gyro system and its performance in the control loop of an orbiting vehicle. The response of the gyro is first studied as a separate unit, but under the influence of gimbal angle and torquing terms. After the behavior of this isolated orbiting system has been described, the response of the over-all system is determined. Finally, a comparison of the vehicle behavior under command of the two-degrees-of-freedom gyro is compared to that under command of the roll-rate gyro system.

NOTATION[1]

Reference Frames

All of the following are orthonormal right-hand sets.

$\bar{b}_k$ — vehicle principal axes: $\bar{b}_1$ roll axis, $\bar{b}_2$ pitch axis, $\bar{b}_3$ yaw axis (For an ideal stabilization system $\bar{b}_1 = \bar{O}_1$, $\bar{b}_2 = \bar{O}_2$, and $\bar{b}_3 = \bar{O}_3$.)

$\bar{g}_{i_k}$ — inner gimbal axes, $\bar{g}_{i_1} = \bar{g}_{o_1}$

$\bar{g}_{o_k}$ — outer gimbal axes, $\bar{g}_{o_3} = \bar{b}_3$

$\bar{O}_k$ — orbital axes system ($\bar{O}_3$ is in the orbital plane and directed radially toward the earth's center from the vehicle center of mass, $\bar{O}_2$ is normal to the orbital plane, and $\bar{O}_1$ is in the direction of flight path for a circular orbit.)

$\bar{r}_k$ — rotor axes, $\bar{r}_2 = \bar{g}_{i_2}$

System Variables

All angles correspond to right-hand rotation.

C_i, C_o — gyro damping coefficient of inner gimbal and outer gimbal, respectively

C_r — roll-rate gyro damping coefficient

h — rotor momentum about its spin axis, $\bar{r}_2$ for both the two-degrees-of-freedom gyroscope and roll-rate gyroscope, $I_{2r}\dot{\theta}_r$

I_k — principal moments of inertia of satellite about satellite center of mass as measured in the $\bar{b}$ reference frame

[1]Refer to figure 1.

$I_{k_{g_i}}$	principal moments of inertia of gyro inner gimbal about gimbal mass center as measured in the $\bar{g}_i$ reference frame
$I_{k_{g_o}}$	principal moments of inertia of gyro outer gimbal about gimbal mass center as measured in the $\bar{g}_o$ reference frame
I_{k_r}	principal moments of inertia of gyro rotor about gyro rotor mass center as measured in the $\bar{r}$ reference frame
I_φ, I_ψ	moments of inertia of the roll and yaw reaction wheels, respectively, about their axis of rotation
K_1	signal gain on roll reaction wheel
K_2	signal gain on gyro spring
K_3	signal gain on yaw reaction wheel
k_f	spring constant of roll-rate gyroscope
M_{TG}	control torque to inner gimbal
S	Laplace operator
T_{m_k}, T_n, T_d	time constants associated with reaction wheel control systems
W_φ	angular speed of the roll reaction wheel about $\bar{b}_1$
W_ψ	angular speed of the yaw reaction wheel about $\bar{b}_3$
θ_g	angle orienting roll-rate gyro gimbal with respect to its case
θ_i	angle orienting the inner gimbal with respect to the outer gimbal (angle between vectors $\bar{g}_{i_2}$ and $\bar{g}_{o_2}$)
θ_o	angle orienting the outer gimbal with respect to the body (angle between vectors $\bar{g}_{o_1}$ and $\bar{b}_1$)
θ_r	angle orienting the rotor with respect to the inner gimbal (angle between vectors $\bar{r}_1$ and $\bar{g}_{i_1}$)
ψ, θ, φ	Euler angles orienting $\bar{b}$ frame with respect to $\bar{O}$ frame (Order of rotation is as shown.)
ω_{k_b}	components (scalar) of the angular velocity in inertial space of the vehicle as measured in the $\bar{b}$ reference frame
$\omega_{k_{g_i}}$	components (scalar) of the angular velocity in inertial space of the inner gimbal as measured in the $\bar{g}_i$ reference frame
$\omega_{k_{g_o}}$	components (scalar) of the angular velocity in inertial space of the outer gimbal as measured in the $\bar{g}_o$ reference frame

ω_{k_r} components (scalar) of the angular velocity in inertial space of the rotor as measured in the $\bar{r}$ reference frame

ω_o scalar magnitude of the inertial angular velocity of the $\bar{O}$ frame (The vehicle orbit is planar.)

Subscripts

b vehicle principal axes

g_i inner gimbal axes

g_o outer gimbal axes

k=1,2,3 axes system components

o orbital axes system

r rotor axes

ANALYSIS OF A TWO-DEGREES-OF-FREEDOM GYROSCOPE AS A SATELLITE YAW ATTITUDE SENSOR

Consider a two-degrees-of-freedom gyroscope mounted in a circular orbiting vehicle. The gyro does not affect the body by its momentum, nor does it send any information to the vehicle's control system. The gyro is, however, susceptible to vehicle motion and therefore may indicate yaw attitude. Assuming small angles and the momentum contribution of the rotor and gimbals inertia to be negligible (ref. 1), one obtains from the appendix equation (A19)

$$h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o)\bar{g}_{i_1} = \bar{O} \tag{1}$$

Equation (1) is for a free inner gimbal. The first term on the left is the input to the inner gimbal axis, which is the vehicle inertial yaw velocity. The second term is an inertial coupling developed from the component of orbital rate along the inner gimbal axis. The third term is due to gyro precession and couples outer gimbal motion into the inner gimbal dynamics.

Equation (A28) gives the expression of a free outer gimbal:

$$-h(\omega_{1_b} - \omega_o\theta_o + \dot{\theta}_i)\bar{g}_{o_3} = \bar{O} \tag{2}$$

A similar explanation of the terms as given before applies here. Linearization eliminates pitch terms from the gyro equations.

Equations (1) and (2) describe a free gyro. These equations may be easily solved, and θ_i shown to indicate φ and θ_o to indicate ψ if the gyro is initially alined. The device represented by the equations is hypothetical and useless for control since it is undamped, so damping proportional to gimbal rotational rate is introduced. Also, the frequency of the gyro oscillation is at orbital frequency. This characteristic is undesirable since the gyro should respond rapidly to vehicle dynamics, enabling the gyro to establish a position in space which can then supply information to the control system. These frequencies can be separated by an electrical spring placed between the inner and outer gimbals. However, orbital coupling is such that θ_o is not only a function of ψ, but also of φ. Therefore, a method is needed for decoupling the roll and yaw modes. This decoupling can be realized if the inner gimbal is constrained to lie in the plane of the local horizon. For this it is necessary that the inner gimbal be torqued as a function of its position with respect to the local vertical. If an electrical spring were used in combination with a vertical reference the signal to a gimbal torquing motor would be

$$M_{TG} = -K_2(\varphi + \theta_i) \tag{3}$$

where K_2 is the spring constant. With the inner gimbal in the plane of the local horizon, the outer gimbal provides a yaw reference. The presence of the roll sensor contributes little to stability for small yaw angles but does affect the transient behavior.

The torquing term (eq. (3)) and inner and outer gimbal damping, C_i and C_o, respectively, are included in the final gyro equations

$$h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o) = -C_i\dot{\theta}_i - K_2(\theta_i + \varphi) \tag{4}$$

and

$$-h(\omega_{1_b} - \omega_o\theta_o + \dot{\theta}_i) = -C_o\dot{\theta}_o \tag{5}$$

The solutions to equations (4) and (5) for θ_i and θ_o are

$$\theta_i = \frac{(-hS^2 - h\omega_oC_oS - K_2C_oS - h^2\omega_o^2 - h\omega_oK_2)\varphi - hC_oS^2\psi}{(C_iC_o + h^2)S^2 + (h\omega_oC_i + h\omega_oC_o + K_2C_o)S + (h^2\omega_o^2 + h\omega_oK_2)} \tag{6}$$

and

$$\theta_o = \frac{hC_iS^2\varphi + (-h^2S^2 - hC_i\omega_oS - h^2\omega_o^2 - hK_2\omega_o)\psi}{(C_iC_o + h^2)S^2 + (h\omega_oC_i + h\omega_oC_o + K_2C_o)S + (h^2\omega_o^2 + h\omega_oK_2)} \tag{7}$$

Several significant results are evident from equations (6) and (7). The spring appreciably changes the frequency of the gyro system to greater than orbital frequency. Outer gimbal damping is very critical to gyro damping since the spring constant will generally be much larger than $h\omega_o$. The term C_iC_o in the coefficient of the S^2 term in the denominator of equation (7) is negligible compared to h^2 for small C_o. An examination of equations (6) and (7) shows that in the steady-state conditions, if higher order effects are neglected, θ_i, the inner gimbal angle, indicates φ, and θ_o, the outer gimbal angle, indicates ψ.

ANALYSIS OF A SATELLITE ATTITUDE CONTROL SYSTEM INCORPORATING A TWO-DEGREES-OF-FREEDOM GYROSCOPE

Description of Physical System

The study will assume that a vehicle is orbiting in a circular, planar, and nonprecessing orbit. It is required that the vehicle be earth pointed at all times. Roll and pitch positions can be sensed and controlled by signals from horizon scanners. Yaw position will be determined by a two-degrees-of-freedom gyroscope as explained in the previous section. Vehicle control is accomplished by transmitting vehicle momentum to reaction wheels where it is then dissipated in the orbit by gravity torques. The speed of the reaction wheels are controlled by the vehicle's attitude relative to the orbit.

The Equations of Motion

The equations of motion of an orbiting body are given in references 2 and 3. The equations of motion of the vehicle are linearized with small-angle approximations

$$\sin\theta = \theta$$

$$\cos\theta = 1$$

and higher order products are considered negligible as is the contribution of gyro momentum to the total vehicle momentum. Under these conditions the pitch equation for the vehicle is uncoupled from the system and need not be considered. The linearized equations are as follows:

$$I_1\dot{\omega}_{1_b} = -3\omega_o^2(I_2 - I_3)\varphi - \omega_o(I_2 - I_3)\omega_{3_b} - I_\varphi\dot{W}_\varphi + \omega_o I_\psi W_\psi \tag{8}$$

$$I_3\dot{\omega}_{3_b} = \omega_o(I_2 - I_1)\omega_{1_b} - \omega_o I_\varphi W_\varphi - I_\psi\dot{W}_\psi \tag{9}$$

where

$$\omega_{1_b} = \dot{\varphi} - \omega_o\psi \tag{10}$$

$$\omega_{3_b} = \dot{\psi} + \omega_o\varphi \tag{11}$$

Equation (8) describes the dynamics of the vehicle about its roll axis. The first term on the right side of equation (8) represents gravity torques. The second is an inertial coupling moment developed from vehicle yaw rate and the orbital rate. The last two terms are due to the effect of reaction wheel torques associated with the control system of the vehicle. Equation (9) describes the dynamics of the vehicle about its yaw axis. The first term on the right side of equation (9) represents an inertia coupling developed from vehicle roll rate and the orbital rate. The last two terms are due to the effect of reaction wheel torques associated with the control system of the vehicle. Equations (10) and (11) define the inertial angular velocities of the vehicle in terms of Euler angles where ω_o is chosen in the negative sense.

The part of the vehicle control system considered here consists of two reaction wheels, one mounted with its spin axis alined with the body roll axis and the other with its spin axis alined with the body yaw axis. The speed of the roll wheel is controlled by a roll position sensor. The speed of the yaw wheel will be controlled by θ_o, the angle orienting the outer gimbal with respect to the vehicle. The equations describing the vehicle control system are as follows:

$$W_\varphi = K_1 \frac{T_n S + 1}{T_d S + 1} \frac{1}{T_m S + 1} \varphi \tag{12}$$

$$W_\psi = -K_3 \frac{1}{T_{m_3} S + 1} \theta_o \tag{13}$$

Equation (12) describes the roll control system and equation (13) describes the yaw control system. The symbols T_m represent the time constants of the motors associated with the reaction wheels; T_n and T_d, the parameters of an ordinary passive lead-lag network; and K the gains of the control system.

At this point it is convenient to summarize the equations which will be the basis for study in this report.

$$I_1 \dot{\omega}_{1_b} = -3\omega_o^2 (I_2 - I_3)\varphi - \omega_o (I_2 - I_3)\omega_{3_b} - I_\varphi \dot{W}_\varphi + \omega_o I_\psi W_\psi \tag{14}$$

$$I_3 \dot{\omega}_{3_b} = \omega_o (I_2 - I_1)\omega_{1_b} - \omega_o I_\varphi W_\varphi - I_\psi \dot{W}_\psi \tag{15}$$

$$W_\varphi = K_1 \frac{T_n S + 1}{T_d S + 1} \frac{1}{T_{m_1} S + 1} \varphi \tag{16}$$

$$W_\psi = -K_3 \frac{1}{T_{m_3} S + 1} \theta_o \tag{17}$$

$$h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o) = -C_i\dot{\theta}_i - K_2(\varphi + \theta_i) \tag{18}$$

$$-h(\dot{\theta}_i + \omega_{1_b} - \omega_o\theta_o) = -C_o\dot{\theta}_o \tag{19}$$

A block diagram of equations (14) through (19) is shown in figure 2.

Previously, the two-degrees-of-freedom gyroscope with roll attitude input was analyzed independently of the vehicle control system. This modified gyroscope will now be analyzed when the sensed yaw attitude error is used in the closed loop of the vehicle to control the speed of the yaw reaction wheel. The effect of the modified gyro on over-all system stability and the ability of the gyro to sense yaw will be determined.

Consider equations (14) through (19). We shall determine the characteristic function of this set of equations. A useful and good assumption to be employed in an analytical study of orbiting bodies with inertias on the order of magnitude of the vehicle considered here is to equate roll and pitch inertias or I_1 and I_2 (ref. 4). It should be mentioned that this assumption is used only in the analytical formulations of this section and not in machine computations.

Assuming $I_1 = I_2$, substitute equations (10), (11), (12), and (13) into equations (14) and (15). From the Laplace transform of the resulting equations, the equations may be written in matrix form as follows:

$$\begin{pmatrix} I_1S^2 + 4\omega_o^2(I_2 - I_3) + I_\varphi K_1 \dfrac{T_nS + 1}{T_dS + 1}\dfrac{S}{T_{m_1}S + 1} & -\omega_o I_3 S & 0 & \dfrac{\omega_o I_\psi K_3}{T_{m_3}S + 1} \\ I_3\omega_o S + \omega_o I_\varphi K_1 \dfrac{T_nS + 1}{T_dS + 1}\dfrac{1}{T_{m_3}S + 1} & I_3S^2 & 0 & \dfrac{-I_\psi K_3 S}{T_{m_3}S + 1} \\ h\omega_o + K_2 & hS & C_iS + h\omega_o + K_2 & hS \\ hS & -h\omega_o & hS & -C_oS - h\omega_o \end{pmatrix} \begin{pmatrix} \varphi \\ \psi \\ \theta_i \\ \theta_o \end{pmatrix} = 0 \tag{20}$$

Denoting the coefficient matrix of equation (20) by M, we obtain:

$$(M)\begin{pmatrix} \varphi \\ \psi \\ \theta_i \\ \theta_o \end{pmatrix} = 0 \tag{21}$$

Equation (21) may be mapped to a more convenient form by the following operations

$$\begin{pmatrix} S & \omega_o & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (M) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varphi \\ \psi \\ \theta_i \\ \theta_o \end{pmatrix} = 0 \qquad (22)$$

The product of the third and fourth matrix of equation (22) is the identity matrix which does not alter the coefficient matrix M. The first matrix will therefore have no effect on the product of the remaining matrices since their product is the null matrix. The first three matrices may be combined to form a new coefficient matrix. The last two matrices when combined form a new state variable matrix. Equation (22) becomes

$$\begin{pmatrix} (I_1S^2 + 4\omega_o^2 I_2 - 3\omega_o^2 I_3)S + I_\varphi K_1 \frac{(T_nS + 1)(S^2 + \omega_o^2)}{(T_dS + 1)(T_{m_1}S + 1)} & 0 & 0 & 0 \\ I_3\omega_o S + \omega_o I_\varphi K_1 \frac{T_nS + 1}{T_dS + 1} \frac{1}{T_{m_1}S + 1} & I_3S + \frac{I_\psi K_3}{T_{m_3}S + 1} & 0 & \frac{-I_\psi K_3 S}{T_{m_3}S + 1} \\ h\omega_o + k_2 & 0 & C_iS + h\omega_o + K_2 & hS \\ hS & C_oS & hS & -C_oS - h\omega_o \end{pmatrix} \begin{pmatrix} \varphi \\ \psi \\ \theta_i \\ \psi + \theta_o \end{pmatrix} = 0$$

(23)

The coefficient matrix of equation (23) is also the determinant describing the stability characteristics of the system. Evaluating this determinant we have

$$[H(S) + I_\varphi K_1 G(S)][M(S)N(S) - I_\psi K_3 C_o R(S)] \qquad (24)$$

where

$$\left.\begin{aligned} H(S) &= (I_1S^2 + 4\omega_o^2 I_2 - 3\omega_o^2 I_3)(T_{m_1}S + 1)(T_dS + 1)S \\ G(S) &= (T_nS + 1)(S^2 + \omega_o^2) \\ M(S) &= (I_3T_{m_3}S^2 + I_3S + K_3I_\psi) \\ N(S) &= (C_iC_o + h^2)S^2 + [h\omega_o(C_i + C_o) + K_2C_o]S + h^2\omega_o^2 + h\omega_oK_2 \\ R(S) &= (C_iS + h\omega_o + K_2)S \end{aligned}\right\} \qquad (25)$$

The terms within the brackets on the left side of expression (24) are related, with the exception of the pitch and yaw vehicle inertias, to the parameters of the vehicle roll dynamics. The terms within the brackets on the right are related to the parameters of the vehicle yaw dynamics and the gyro dynamics.

Thus it appears that equating the roll and pitch inertias has effectively separated the system roll and yaw-gyro modes. The roll mode is of little interest here. A more detailed discussion of this mode is available in reference 4. Only the yaw-gyro mode is of interest here.

If $C_o = 0$, the second factor in expression (24) can be separated into two factors which describe the stability characteristics of the yaw mode and the gyro mode since each factor contains only terms relating to the stability of its own mode. Therefore, C_o produces a coupling effect between the vehicle and gyro, as would be expected.

The effect of outer gimbal damping, C_o, can be demonstrated by a numerical example. Assume the following values in which the vehicle inertias and altitude are typical of the Nimbus meteorological satellite:

$T_{m_3} = 38.5$ sec	$h = 44.25\times10^{-4}$ slug-ft^2/sec
$I_3 = 120$ slug-ft^2	$\omega_o = 0.9725\times10^{-3}$ radian/sec
$C_i = 22.13\times10^{-4}$ slug-ft^2/sec	$I_\psi = 0.00212$ slug-ft^2
$K_2 = 22.14\times10^{-4}$ slug-ft^2/sec^2	$K_3 = 1000$ sec^{-1}
	$C_o = 0$ and 0.44×10^{-4} slug-ft^2/sec

For the first case, C_o is chosen as 0. This choice uncouples the yaw and gyro modes. For the second case C_o is chosen 1/50 the size of C_i. This choice introduces slight coupling between the two modes and K_2 is chosen sufficiently large to emphasize the effect of C_o. The term $C_iC_o << h^2$.

Decoupled case ($C_o = 0$).- Here one has simply:

$$[S^2 + (1/T_{m_3})S + (K_3I_\psi/I_3T_{m_3})]\,[S^2 + (\omega_oC_i/h)S + (h^2\omega_o^2 + K_2\omega_o/h)] \qquad (26)$$

or when numerical values are substituted

$$\overbrace{(S^2 + 0.026S + 0.46\times10^{-3})}^{\text{Yaw mode}}\ \overbrace{(S^2 + 0.49\times10^{-3} + 0.49\times10^{-3})}^{\text{Gyro mode}} \qquad (27)$$

Consider the gyro term. The damping ratio and frequency are

$$\rho = 0.0111 \text{ and } \omega_n = 2.2\times10^{-2} \text{ radian/sec}$$

Coupled case ($C_o = 0.44\times10^{-4}$).- Substituting numerical values in the yaw-gyro term of expression (24), one has:

$$S^4 + 3.13\times10^{-2}S^3 + 1.08\times10^{-3}S^2 + 0.1485\times10^{-4}S + 222\times10^{-4} \qquad (28)$$

One obtains upon factoring expression (28):

$$\overbrace{(S^2 + 0.026S + 0.46\times10^{-3})}^{\text{Yaw mode}}\overbrace{(S^2 + 0.53\times10^{-2}S + 0.48\times10^{-3})}^{\text{Gyro mode}} \qquad (29)$$

The yaw mode remains unchanged. The frequency of the gyro mode is unchanged, but the damping is increased by a factor of 10.

The analog time histories in figures 3 to 5 illustrate the transient behavior of the vehicle when the two-degrees-of-freedom gyro is used in the yaw control loop. In each set of time histories the first two describe vehicle motion with respect to the orbital axes. The second two describe the indicating performance of the gyro. Since the inner gimbal angle is supposed to indicate the negative of roll position and the outer gimbal the negative of yaw position, the quantities $\varphi + \theta_i$ and $\psi + \theta_o$ should become zero as rapidly as possible.

Figure 3 is the time history for the gyro with no outer gimbal damping but with a light spring. This case is identical to the decoupled case discussed above except the spring is one-tenth as large. Figure 4 illustrates the decoupled case discussed above. This system shows considerable improvement over the system with a light spring. Figure 5 illustrates the coupling case previously discussed. This introduces outer gimbal damping to the gyro and again considerably improves over-all system performance. The value of inner gimbal damping was the same for the three figures. The time histories use the values of vehicle inertia given in table I and do not equate roll and pitch inertias. Analog studies in which the inertia values in table I are used correspond well to the analytical analysis which equates the roll and pitch inertias.

An examination of expression (24) shows that an undamped gyro mode exists if there is no damping on either the inner or outer gimbal and that the gyro mode can be damped if only inner gimbal damping is provided. Utilizing Routh's criterion one can also show that it is possible to stabilize the system with outer gimbal damping only. Equations (27) and (29) indicated the need for damping on both gimbals since the gyro mode was poorly damped with inner gimbal damping only.

Within a given range of control system gains the effect of gyro parameters can be studied by considering the denominator of equation (6). From this equation it is readily seen that C_o has an important effect on gyro damping and has the effect of coupling K_2 into the damping term with negligible effect on the frequency of the gyro mode. Small increases of C_o will greatly improve stability characteristics. However, using C_o to improve system performance makes the gyro system more responsive to body motions. Thus C_o, while having advantages, does have limitations and cannot be freely varied.

The indicating ability of the gyro is a direct function of the ratio of the gyro spring constant to the vehicle control system gain. The high-frequency motion of the vehicle induced by a high gain control system cannot be indicated by the gyroscope. This oscillation, however, manifests itself for only a relatively short duration and the frequency of the vehicle dynamics is then approximately orbital frequency which is easily indicated by the gyro.

DISCUSSION

The preceding portions of the report have been concerned with the analysis of a two-degrees-of-freedom gyro as a satellite yaw sensor and its subsequent use in the yaw control system of an orbiting vehicle. There are several items relative to the two-degrees-of-freedom gyroscope which should be mentioned and briefly discussed.

The Two-Degrees-of-Freedom Gyroscope as a Satellite Yaw Reference

The two-degrees-of-freedom gyro has been shown to be a satisfactory yaw sensor. However, the analysis was confined to small-angle measurements from the orbital plane. Since it is occasionally desirable to slew an orbiting earth-pointed vehicle to a yaw position offset, the effect of large yaw angles on system stability and the ability of the gyro to indicate yaw attitude error are of interest.

It can be shown analytically that the gyro will indicate the position error of a positionally offset vehicle for large vehicle yaw angles. It is necessary to determine whether stability is maintained at large offset angles.

The values given in table I were used with $C_i = 22.13 \times 10^{-4}$, $C_o = 0.44 \times 10^{-4}$, and $K_2 = 22.14 \times 10^{-4}$ to study the stability of the vehicle for offset angles ranging from 0^o to 90^o. The pitch mode was stabilized by a control system identical to the roll control system with pitch position controlling the reaction wheel speed. The system remained stable without a modified motor torque generator for values of offset angles as great as 60^o. Offset angles greater than 60^o would require that the motor torque generator signal be modified by the introduction of a separate pitch signal to compensate for pitch error developed by the offset.

A Comparison Between the Performance of a Two-Degrees-of-Freedom Gyroscope and a Roll-Rate Gyroscope

It has been proposed that a roll-rate gyroscope could be used in orbiting earth-pointed vehicles to supply yaw attitude information to the yaw control system. A block diagram of a vehicle using a roll-rate gyro is shown in figure 6. A complete study of the performance of the roll-rate gyro in an orbiting vehicle is available in reference 4. The performance of the proposed two-degrees-of-freedom gyro can be compared with a roll-rate gyro. Figure 7 illustrates the performance of a roll-rate gyro system which can be compared with the two-degrees-of-freedom gyro system in figures 3 through 5. For the comparison, the total gain on yaw position error obtained from the roll-rate signal for the roll-rate gyro was equal to the gain on the outer gimbal angle used to control the yaw control system for the two-degrees-of-freedom gyro or in equation form

$$\text{Roll-rate gyro gain} = K_3 \frac{h\omega_o + k_f}{h\omega_o} \qquad (30)$$

The rotor momentum of the roll-rate gyro is the same as that of the two-degrees-of-freedom gyro. The ratio C_r/h for the roll-rate gyro is 0.5. The two-degrees-of-freedom gyro shows a definite improvement over the roll-rate gyro. The difficulty with the roll-rate gyro system is that it produces primarily a roll-rate signal in combination with a yaw-position signal caused by coupling. The presence of the large roll-rate signal from the roll-rate gyro greatly decreases the performance of this system. The two-degrees-of-freedom gyro system, however, essentially nullifies the effect of the roll-rate term by converting roll position to roll rate where it cancels the roll-rate signal from the gyro. The roll-rate gyro system illustrated in figure 7 has large overshoots and is considerably less damped than the two-degrees-of-freedom gyro system.

Drift

The causes of drift are many (ref. 5), for example, gravity torques, solar torques, and gyro structural irregularities. A detailed study of the sources and mathematical description of drift are beyond the scope of this report but its effect on the steady-state conditions of the vehicle is of interest.

Assume that the gyro experiences a drift-producing torque, thus producing an unknown offset in combination with the desired signal. The effect on the steady-state position of the vehicle axis with respect to the orbital axis can be determined from the equations at steady state. Let $h\dot{\theta}_{d_i}$ represent a constant torque on the inner gimbal and $h\dot{\theta}_{d_o}$ a constant torque on the outer gimbal. The steady-state equations of equations (14) through (19) with the addition of the constant gyro torques become

$$\left.\begin{aligned}
0 &= -4\omega_o^2(I_2 - I_3)\varphi + \omega_o I_\psi W_\psi \\
0 &= -\omega_o^2(I_2 - I_1)\psi - \omega_o I_\varphi W_\varphi \\
0 &= -\omega_o\varphi - \omega_o\theta_i + K_2/h(-\theta_i - \varphi) + \dot{\theta}_{d_i} \\
0 &= \omega_o\psi + \omega_o\theta_o + \dot{\theta}_{d_o} \\
W_\varphi &= K_1\varphi \\
W_\psi &= -K_3\theta_o
\end{aligned}\right\} \qquad (31)$$

Solving equations (31) for φ_{ss} and ψ_{ss}, the steady-state values of roll and yaw positions, respectively, one obtains

$$\varphi_{ss} \cong \frac{I_1 - I_2}{I_\varphi K_1}\dot{\theta}_{d_o} \text{ and } \psi_{ss} \cong \frac{\dot{\theta}_{d_o}}{\omega_o}$$

Observe that these values are independent of the torque on the inner gimbal angle. Assuming a torque producing a gimbal rate of 2°/hr and using the vehicle parameters given in table I, one obtains the following steady-state errors.

$$\varphi_{ss} = 0.01^\circ \text{ and } \psi_{ss} = 0.55^\circ$$

It should be mentioned that the same drift-producing torque assumed here when applied to a roll-rate gyroscope produces an identical yaw attitude offset.

CONCLUSIONS

1. Use of a two-degrees-of-freedom gyro and a vertical sensor provides adequate information for three axis stabilization and control of a vertically oriented satellite. The system allows the heading for yaw of the vehicle to be maintained at any desired value.

2. If yaw angles less that 60° from the orbital plane are desired, vehicle pitch compensation is not required for control of the gyro's inner gimbal.

3. As a yaw nulling device, a system using the two-degrees-of-freedom gyro gives performance superior to that of a system using a roll-rate gyro.

4. System performance is an important function of the inner and outer gimbal damping (C_i and C_o, respectively) and gyro rotor momentum, h. If the damping and momentum are small, it is necessary to consider the high frequency modes which are otherwise negligible. If the gyro momentum, h, becomes too large, the gyro will become a source of torque to the vehicle as well as a sensor. The damping and momentum should be selected to avoid these extremes.

5. The sensing ability of the two-degrees-of-freedom gyroscope is improved by increasing the frequency of the gyro relative to the frequency of the vehicle.

6. The effect of a drift-producing torque on the two-degrees-of-freedom gyro system is the same as on a roll-rate gyro system.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Oct. 23, 1963

APPENDIX

DERIVATION OF EQUATIONS OF MOTION OF A TWO-DEGREES-OF-FREEDOM GYROSCOPE

(The derivation presented here is substantially based upon unpublished methods compiled at Ames Research Center by J. S. Pappas, K. C. Grover, and V. K. Merrick.)

Using right-hand orthonormal systems and referring to figure 1, we may define the relationships between the components of the gyro and the body. Consider the rotation of a coordinate system, $\bar{r}$, fixed in the rotor, with respect to a coordinate system, $\bar{g}_i$, fixed in the inner gimbal. This rotation may be defined by a matrix:

$$\begin{pmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \end{pmatrix} = \begin{pmatrix} c\theta_r & 0 & -s\theta_r \\ 0 & 1 & 0 \\ s\theta_r & 0 & c\theta_r \end{pmatrix} \begin{pmatrix} \bar{g}_{i_1} \\ \bar{g}_{i_2} \\ \bar{g}_{i_3} \end{pmatrix} \tag{A1}$$

where s = sine and c = cosine. The velocity relationship of the rotor and inner gimbal with respect to inertial space is

$$\bar{\omega}_r = \bar{\omega}_{g_i} + \dot{\theta}_r \bar{g}_{i_2} \tag{A2}$$

where

$\bar{\omega}_r$ vector inertial angular velocity of the rotor

$\bar{\omega}_{g_i}$ vector inertial angular velocity of the inner gimbal

$\dot{\theta}_r \bar{g}_{i_2}$ vector angular velocity of the rotor relative to the inner gimbal

The inner gimbal is free to rotate with respect to an outer gimbal. If $\bar{g}_o$ is a coordinate system fixed in the outer gimbal, one has

$$\begin{pmatrix} \bar{g}_{i_1} \\ \bar{g}_{i_2} \\ \bar{g}_{i_3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\theta_i & s\theta_i \\ 0 & -s\theta_i & c\theta_i \end{pmatrix} \begin{pmatrix} \bar{g}_{o_1} \\ \bar{g}_{o_2} \\ \bar{g}_{o_3} \end{pmatrix} \tag{A3}$$

where the velocity relationship of the two frames with respect to inertial space is

$$\bar{\omega}_{g_i} = \bar{\omega}_{g_o} + \dot{\theta}_i \bar{g}_{o_1} \tag{A4}$$

where

$\bar{\omega}_{g_o}$ vector inertial angular velocity of the outer gimbal

$\dot{\theta}_i \bar{g}_{o_1}$ angular velocity of the inner gimbal relative to the outer gimbal

Finally, the outer gimbal is free to rotate with respect to the body. This rotation is defined by the matrix:

$$\begin{pmatrix} \bar{g}_{o_1} \\ \bar{g}_{o_2} \\ \bar{g}_{o_3} \end{pmatrix} = \begin{pmatrix} c\theta_o & s\theta_o & 0 \\ -s\theta_o & c\theta_o & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{pmatrix} \tag{A5}$$

where the velocity relationship of the outer gimbal and body with respect to inertial space is

$$\bar{\omega}_{g_o} = \bar{\omega}_b + \dot{\theta}_o\bar{b}_3 \tag{A6}$$

where

$\bar{\omega}_b$ vector inertial angular velocity of the body

$\dot{\theta}_o\bar{b}_3$ vector angular velocity of the outer gimbal relative to the body

It is desirable to summarize the scalar relationships implied by the previous work. The scalar components of the inertial velocities of the inner and outer gimbal are expressed in terms of the scalar components of the inertial velocity of the body. Written in matrix form they are

$$\begin{pmatrix} \omega_{1g_i} \\ \omega_{2g_i} \\ \omega_{3g_i} \end{pmatrix} = \begin{pmatrix} c\theta_o & s\theta_o & 0 \\ -c\theta_i s\theta_o & c\theta_i c\theta_o & s\theta_i \\ -s\theta_i s\theta_o & -s\theta_i c\theta_o & c\theta_i \end{pmatrix} \begin{pmatrix} \omega_{1_b} + \dot{\theta}_i c\theta_o \\ \omega_{2_b} + \dot{\theta}_i s\theta_o \\ \omega_{3_b} + \dot{\theta}_o \end{pmatrix} \tag{A7}$$

and

$$\begin{pmatrix} \omega_{1g_o} \\ \omega_{2g_o} \\ \omega_{3g_o} \end{pmatrix} = \begin{pmatrix} c\theta_o & s\theta_o & 0 \\ -s\theta_o & c\theta_o & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \omega_{1_b} \\ \omega_{2_b} \\ \omega_{3_b} + \dot{\theta}_o \end{pmatrix} \tag{A8}$$

where

ω_{1_b} scalar component of the angular velocity in inertial space of the body with respect to its roll axis

ω_{2_b} scalar component of the angular velocity in inertial space of the body with respect to its pitch axis

ω_{3_b} scalar component of the angular velocity in inertial space of the body with respect to its yaw axis

The derivation of the scalar components of the inertial velocity of the body in terms of the Euler angles of the body with respect to the orbit is not given in this report, but is only summarized here for small angle deviations. They are

$$\omega_{1_b} = \dot{\varphi} - \omega_o\psi \tag{A9}$$

$$\omega_{2_b} = \dot{\theta} - \omega_o \tag{A10}$$

$$\omega_{3_b} = \dot{\psi} + \omega_o\varphi \tag{A11}$$

The two-degrees-of-freedom gyro contains three dynamical modes. The first describes the rotation of the rotor with respect to the inner gimbal and contains no information for a constant speed and axial symmetrical rotor. The second is the rotation of the inner gimbal with respect to the outer gimbal. Consider the axis system of the inner gimbal, $\overline{g}_i$, as a reference. The total angular momentum of the rotor and inner gimbal may be expressed in vector form.

$$\overline{H}_{\text{inner gimbal system}} = \overline{H}_{\text{inner gimbal}} + \overline{H}_{\text{rotor}} \tag{A12}$$

Expanding

$$\overline{H}_{IGS} = \overline{\overline{I}}_{g_i} \cdot \overline{\omega}_{g_i} + \overline{\overline{I}}_r \cdot \overline{\omega}_r \tag{A13}$$

Further expansion in terms of matrices and neglecting products of inertia yields

$$\overline{H}_{IGS} = \left(\overline{g}_{i_1}\overline{g}_{i_2}\overline{g}_{i_3}\right)\begin{pmatrix} I_{1_{g_i}} & 0 & 0 \\ 0 & I_{2_{g_i}} & 0 \\ 0 & 0 & I_{3_{g_i}} \end{pmatrix}\begin{pmatrix} \overline{g}_{i_1} \\ \overline{g}_{i_2} \\ \overline{g}_{i_3} \end{pmatrix}\cdot\left(\overline{g}_{i_1}\overline{g}_{i_2}\overline{g}_{i_3}\right)\begin{pmatrix} \omega_{1g_i} \\ \omega_{2g_i} \\ \omega_{3g_i} \end{pmatrix}$$

$$+ \left(\overline{r}_1\overline{r}_2\overline{r}_3\right)\begin{pmatrix} I_{1_r} & 0 & 0 \\ 0 & I_{2_r} & 0 \\ 0 & 0 & I_{3r} \end{pmatrix}\begin{pmatrix} \overline{r}_1 \\ \overline{r}_2 \\ \overline{r}_3 \end{pmatrix}\cdot\left(\overline{r}_1\overline{r}_2\overline{r}_3\right)\begin{pmatrix} \omega_{1_r} \\ \omega_{2_r} \\ \omega_{3_r} \end{pmatrix} \tag{A14}$$

Transforming all terms of this equation to the inner gimbal frame, g_i, and assuming $I_{1_r} = I_{3_r}$, one obtains

$$\overline{H}_{IGS} = \left(I_{1_{g_i}} + I_{1_r}\right)\omega_{1_{g_i}}\overline{g}_{i_1} + \left[I_{2_{g_i}}\omega_{2_{g_i}} + I_{2_r}\left(\omega_{2_{g_i}} + \dot{\theta}_r\right)\right]\overline{g}_{i_2}$$

$$+ \left(I_{3_{g_i}} + I_{3_r}\right)\omega_{3_{g_i}}\overline{g}_{i_3} \tag{A15}$$

The time derivative of this expression is

$$\dot{\overline{H}}_{IGS\overline{g}_{i_1}\text{axis}} = \left[\left(I_{1g_i} + I_{1r}\right)\dot{\omega}_{1g_i} + \left(I_{3g_i} - I_{2g_i} + I_{3r} - I_{2r}\right)\omega_{2g_i}\omega_{3g_i} - h\omega_{3g_i}\right]\overline{g}_{i_1}$$

$$= \begin{bmatrix}\text{External torques}\\ \text{about } g_{i_1} \text{ axis}\end{bmatrix}\overline{g}_{i_1} \qquad \text{(A16)}$$

Only the $\overline{g}_{i_1}$ component need be considered since the $\overline{g}_{i_2}$ and $\overline{g}_{i_3}$ axes have no gimbaled freedom. If the momentum contribution of the gimbal and rotor inertias are negligible and small angles are assumed, one has upon substituting the value of ω_{3g_i} from equation (A7) into (A16)

$$-h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o)\overline{g}_{i_1} = \begin{bmatrix}\text{External torques}\\ \text{about } g_{i_1} \text{ axis}\end{bmatrix}\overline{g}_{i_1} \qquad \text{(A17)}$$

Vehicle stability considerations will indicate the necessity of a negative h or

$$h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o)\overline{g}_{i_1} = \begin{bmatrix}\text{External torques}\\ \text{about } g_{i_1} \text{ axis}\end{bmatrix}\overline{g}_{i_1} \qquad \text{(A18)}$$

If the inner gimbal is free, the right side of equation (A18) is 0 or

$$h(\omega_{3_b} + \omega_o\theta_i + \dot{\theta}_o)\overline{g}_{i_1} = \overline{0} \qquad \text{(A19)}$$

The third dynamic mode is the rotation of the outer gimbal frame with respect to the body. Consider the outer gimbal axis system $\overline{g}_o$ as a reference. The total angular momentum of the rotor, inner gimbal, and outer gimbal may be expressed in vector form:

$$\overline{H}_{\text{outer gimbal system}} = \overline{H}_{\text{rotor}} + \overline{H}_{\text{inner gimbal}} + \overline{H}_{\text{outer gimbal}} \qquad \text{(A20)}$$

Expanding

$$\overline{H}_{OGS} = \overline{\overline{I}}_r \cdot \overline{\omega}_r + \overline{\overline{I}}_{gi} \cdot \overline{\omega}_{gi} + \overline{\overline{I}}_{g_o} \cdot \overline{\omega}_{g_o} \qquad \text{(A21)}$$

Further expansion in terms of matrices and neglecting products of inertia yields

$$\overline{H}_{OGS} = \left(\overline{r}_1\overline{r}_2\overline{r}_3\right) \begin{pmatrix} I_{1_r} & 0 & 0 \\ 0 & I_{2_r} & 0 \\ 0 & 0 & I_{3_r} \end{pmatrix} \begin{pmatrix} \overline{r}_1 \\ \overline{r}_2 \\ \overline{r}_3 \end{pmatrix} \cdot \left(\overline{r}_1\overline{r}_2\overline{r}_3\right) \begin{pmatrix} \omega_{1_r} \\ \omega_{2_r} \\ \omega_{3_r} \end{pmatrix}$$

$$+ \left(\overline{g}_{i_1}\overline{g}_{i_2}\overline{g}_{i_3}\right) \begin{pmatrix} I_{1_{g_i}} & 0 & 0 \\ 0 & I_{2_{g_i}} & 0 \\ 0 & 0 & I_{3_{g_i}} \end{pmatrix} \begin{pmatrix} \overline{g}_{i_1} \\ \overline{g}_{i_2} \\ \overline{g}_{i_3} \end{pmatrix} \cdot \left(\overline{g}_{i_1}\overline{g}_{i_2}\overline{g}_{i_3}\right) \begin{pmatrix} \omega_{1_{g_i}} \\ \omega_{2_{g_i}} \\ \omega_{3_{g_i}} \end{pmatrix}$$

$$+ \left(\overline{g}_{o_1}\overline{g}_{o_2}\overline{g}_{o_3}\right) \begin{pmatrix} I_{1_{g_o}} & 0 & 0 \\ 0 & I_{2_{g_o}} & 0 \\ 0 & 0 & I_{3_{g_o}} \end{pmatrix} \begin{pmatrix} \overline{g}_{o_1} \\ \overline{g}_{o_2} \\ \overline{g}_{o_3} \end{pmatrix} \cdot \left(\overline{g}_{o_1}\overline{g}_{o_2}\overline{g}_{o_3}\right) \begin{pmatrix} \omega_{1_{g_o}} \\ \omega_{2_{g_o}} \\ \omega_{3_{g_o}} \end{pmatrix} \tag{A22}$$

Transforming all terms of this equation to the outer gimbal frame, $\overline{g}_o$, one obtains

$$\begin{aligned}
\overline{H}_{OGS} = &\left[\left(I_{1_{g_o}} + I_{1_{g_i}} + I_{1_r}\right)\omega_{1_{g_o}} + \left(I_{1_{g_i}} + I_{1_r}\right)\dot{\theta}_i\right]\overline{g}_{o_1} \\
&+ \left[I_{2_{g_o}}\omega_{2_{g_o}} + \left(I_{2_{g_i}}c^2\theta_i + I_{3_{g_i}}s^2\theta_i\right)\omega_{2_{g_o}} + \left(I_{2_{g_i}} - I_{3_{g_i}}\right)s\theta_i c\theta_i \omega_{3_{g_o}}\right. \\
&+ \left(I_{2_r}c^2\theta_i + I_{1_r}s^2\theta_i\right)\left(\omega_{2_{g_o}} + \dot{\theta}_r c\theta_i\right) + \left(I_{2_r}s\theta_i c\theta_i - I_{1_r}s\theta_i c\theta_i\right)\Big(\omega_{3_{g_o}} \\
&\left.+ \theta_r s\theta_i\Big)\right]\overline{g}_{o_2} + \left[I_{3_{g_o}}\omega_{3_{g_o}} + \left(I_{2_{g_i}} - I_{3_{g_i}}\right)s\theta_i c\theta_i \omega_{2_{g_o}} + \Big(I_{2_{g_i}}s^2\theta_i\right. \\
&+ I_{3_{g_i}}c^2\theta_i\Big)\omega_{3_{g_o}} + \left(I_{2_r} - I_{1_r}\right)\left(\omega_{2_{g_o}} + \dot{\theta}_r c\theta_i\right)s\theta_i c\theta_i \\
&\left.+ \left(I_{2_r}s^2\theta_i + I_{1_r}c^2\theta_i\right)\left(\omega_{3_{g_o}} + \dot{\theta}_r s\theta_i\right)\right]\overline{g}_{o_3}
\end{aligned} \tag{A23}$$

Assuming small angles one obtains

$$\overline{H}_{OGS} = \left[I_{1_{g_o}} \omega_{1_{g_o}} + \left(I_{1_{g_i}} + I_{1_r} \right) \left(\omega_{1_{g_o}} + \dot{\theta}_i \right) \right] \overline{g}_{o_1}$$

$$+ \left[\left(I_{2_{g_o}} + I_{2_{g_i}} \right) \omega_{2_{g_o}} + I_{2_r} \left(\omega_{2_{g_o}} + \dot{\theta}_r \right) \right] \overline{g}_{o_2}$$

$$+ \left[\left(I_{3_r} + I_{3_{g_i}} + I_{3_{g_o}} \right) \omega_{3_{g_o}} + I_{2_r} \dot{\theta}_r \theta_i \right.$$

$$\left. - \left(I_{2_{g_i}} - I_{3_{g_i}} + I_{2_r} - I_{3_r} \right) \omega_o \theta_i \right] \overline{g}_{o_3} \qquad \text{(A24)}$$

Taking the time derivative with respect to inertial space, one obtains for the $\overline{g}_{o_3}$ component of $\overline{H}_{OGS}$

$$\dot{\overline{H}}_{OGS g_{o_3} \text{axis}} = \left\{ \left(I_{3_r} + I_{3_{g_i}} + I_{3_{g_o}} \right) \dot{\omega}_{3_{g_o}} + \left[h - \left(I_{2_{g_i}} - I_{3_{g_i}} + I_{2_r} - I_{3_r} \right) \omega_o \right] \theta_i \right.$$

$$\left. + \left(I_{2_{g_o}} + I_{2_{g_i}} + I_{2_r} - I_{1_{g_o}} - I_{1_{g_i}} - I_{1_r} \right) \left(-\omega_o \omega_{1_b} + \omega_o^2 \theta_o \right) + h \omega_{1_{g_o}} \right\} \overline{g}_{o_3}$$

$$= \begin{bmatrix} \text{External torques} \\ \text{about } \overline{g}_{o_3} \text{ axis} \end{bmatrix} \overline{g}_{o_3} \qquad \text{(A25)}$$

Only the $\overline{g}_{o_3}$ need by considered since the $\overline{g}_{o_1}$ and $\overline{g}_{o_2}$ axes have no gimbaled freedom. If the momentum contribution of the gimbal and rotor inertias are negligible, one has upon substituting the value of $\omega_{1_{g_o}}$ from equation (A8) into equation (A25)

$$h(\omega_{1_b} - \omega_o \theta_o + \dot{\theta}_i) \overline{g}_{o_3} = \begin{bmatrix} \text{External torques} \\ \text{about } \overline{g}_{o_3} \text{ axis} \end{bmatrix} \overline{g}_{o_3} \qquad \text{(A26)}$$

A negative h changes this equation to

$$-h(\omega_{1_b} - \omega_o \theta_o + \dot{\theta}_i) \overline{g}_{o_3} = \begin{bmatrix} \text{External torques} \\ \text{about } \overline{g}_{o_3} \text{ axis} \end{bmatrix} \overline{g}_{o_3} \qquad \text{(A27)}$$

If the inner gimbal is free, the right side of equation (A27) is 0 or

$$-h(\omega_{1_b} - \omega_o \theta_o + \dot{\theta}_i) \overline{g}_{o_3} = \overline{0} \qquad \text{(A28)}$$

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TABLE OF VALUES

(Vehicle inertias and altitude are typical of the Nimbus meteorological satellite)

$I_1 = 190$ slug-ft^2

$I_2 = 146$ slug-ft^2

$I_3 = 120$ slug-ft^2

$I_\varphi = 0.00212$ slug-ft^2

$I_\psi = 0.00212$ slug-ft^2

$H = 60{,}000$ dyne-cm-sec/radian $= 44.25\times10^{-4}$ slug-ft^2/sec

$\omega_o = 0.9725\times10^{-3}$ radian/sec

$T_n = 7.0$ sec

$T_d = 0.7$ sec

$T_{m_1} = 38.5$ sec

$T_{m_3} = 38.5$ sec

$K_1 = 1000$ sec^{-1}

$K_3 = 1000$ sec^{-1}

Time constant of roll-rate gyro = 0.2 sec

The values of other parameters are stated as needed.

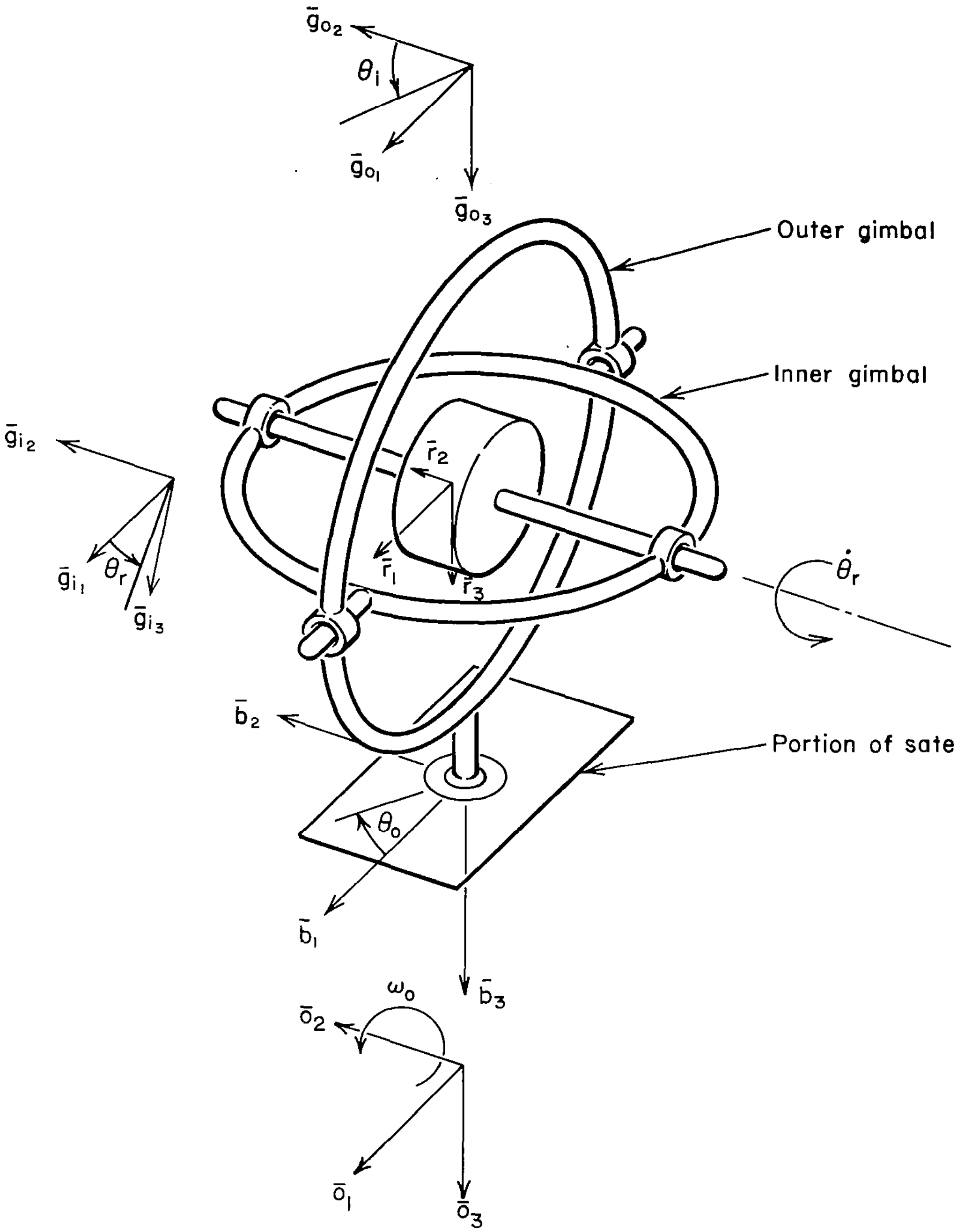


Figure 1.- Two degrees of freedom gyroscope showing position of reference frames and identification of gyro angles. (All axes systems have origin at center of mass of gyro, but are displaced for clarity.)

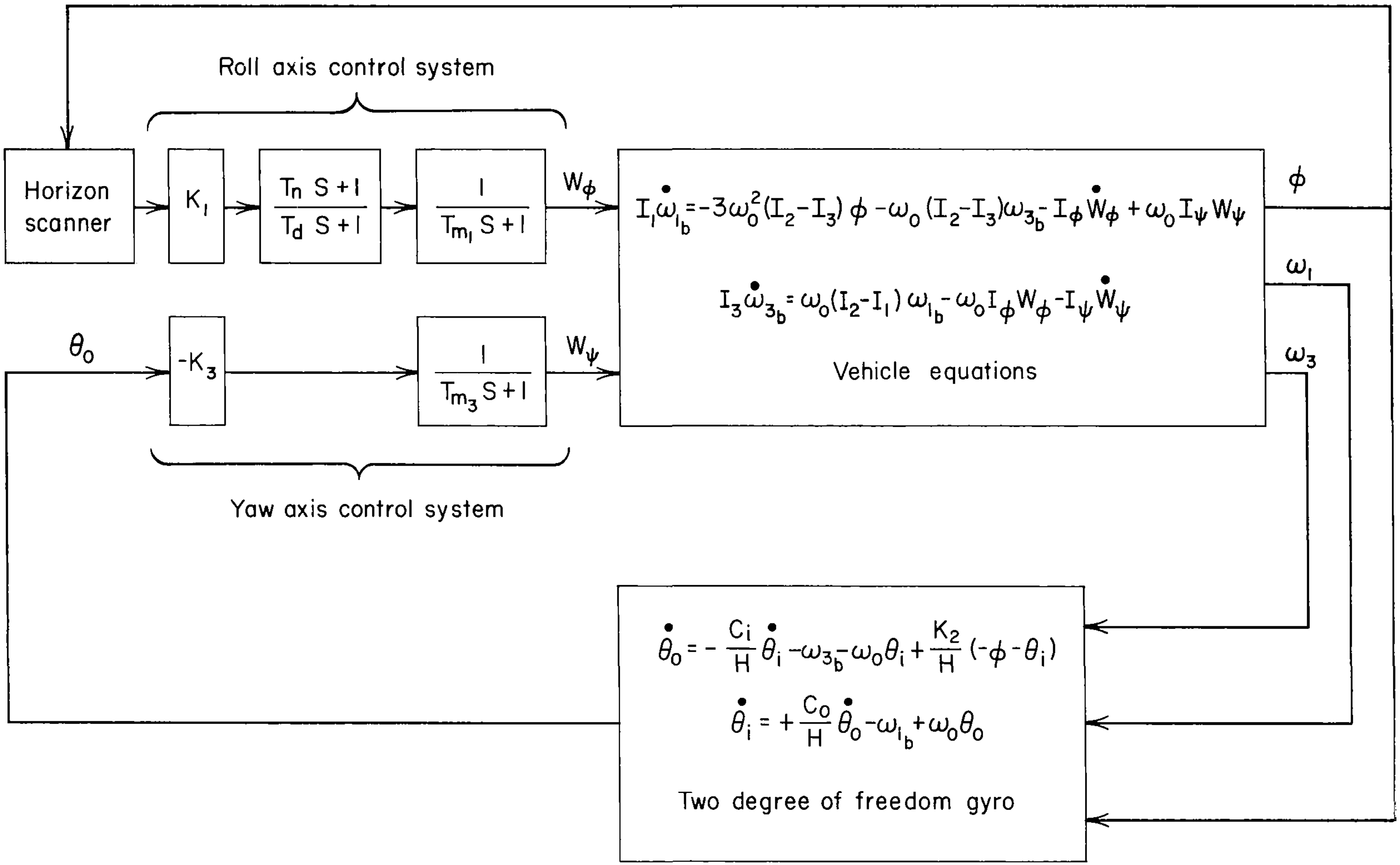
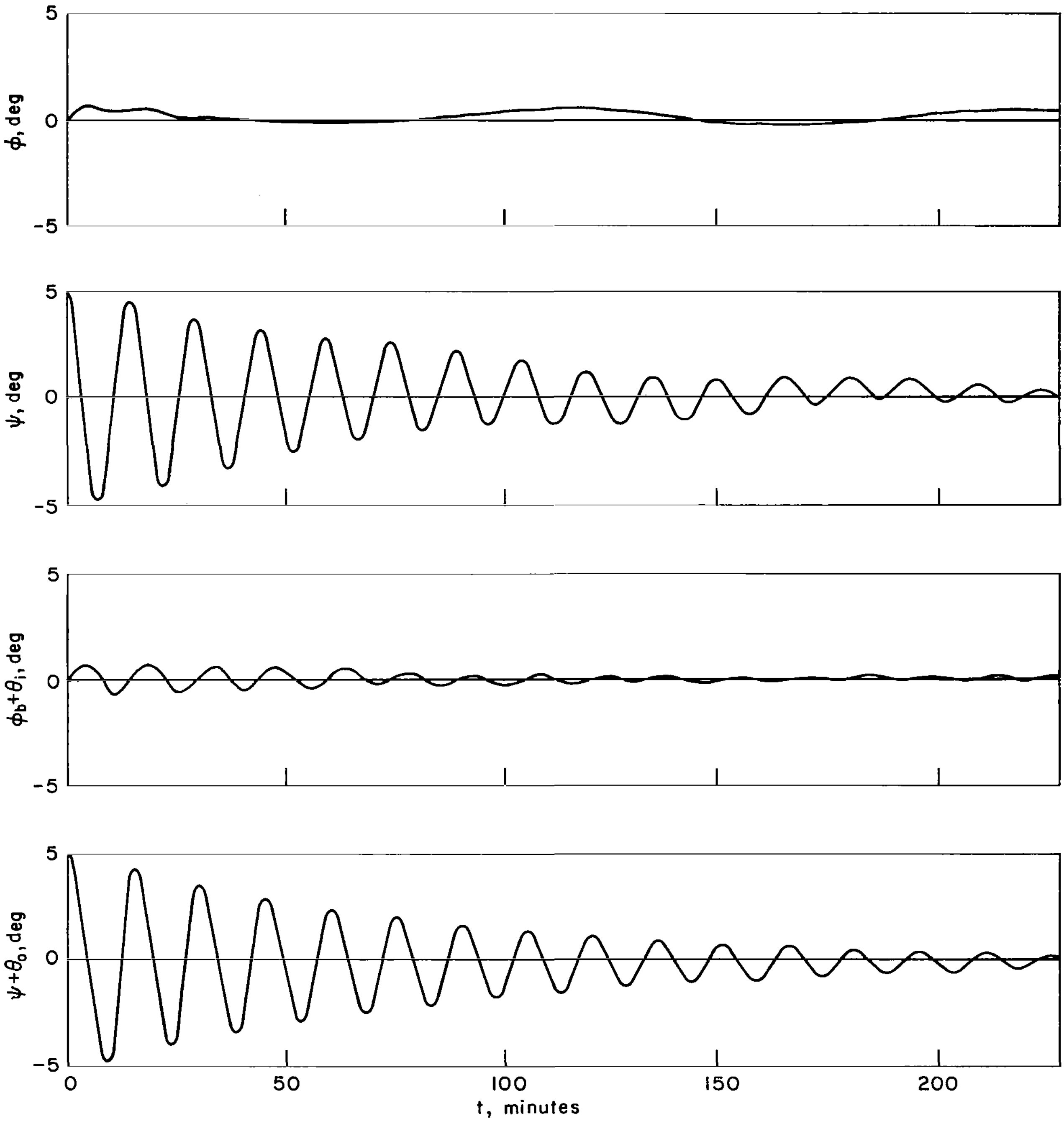
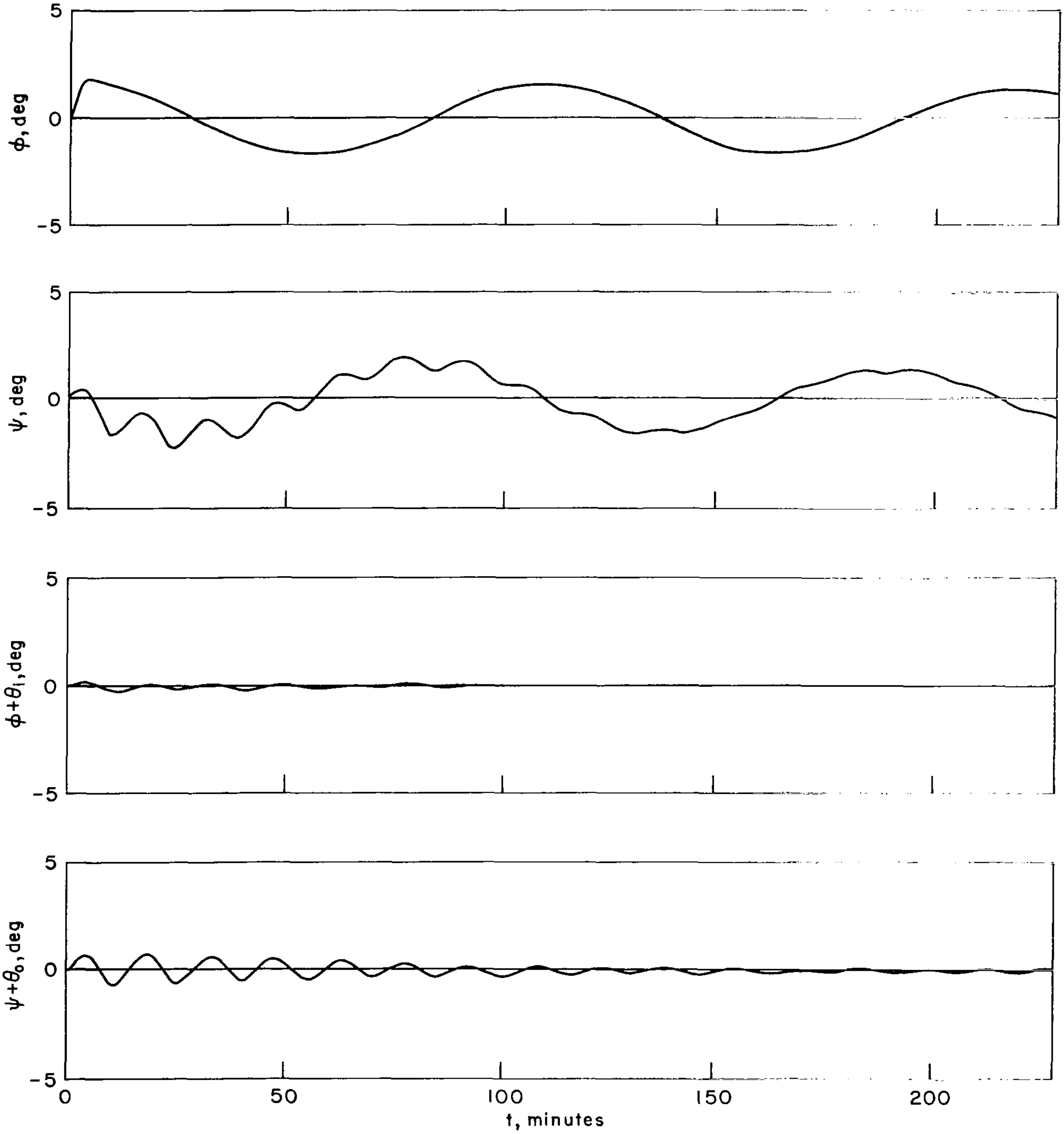


Figure 2.- Block diagram of system and associated control system using a two degrees of freedom gyroscope as a yaw sensor.



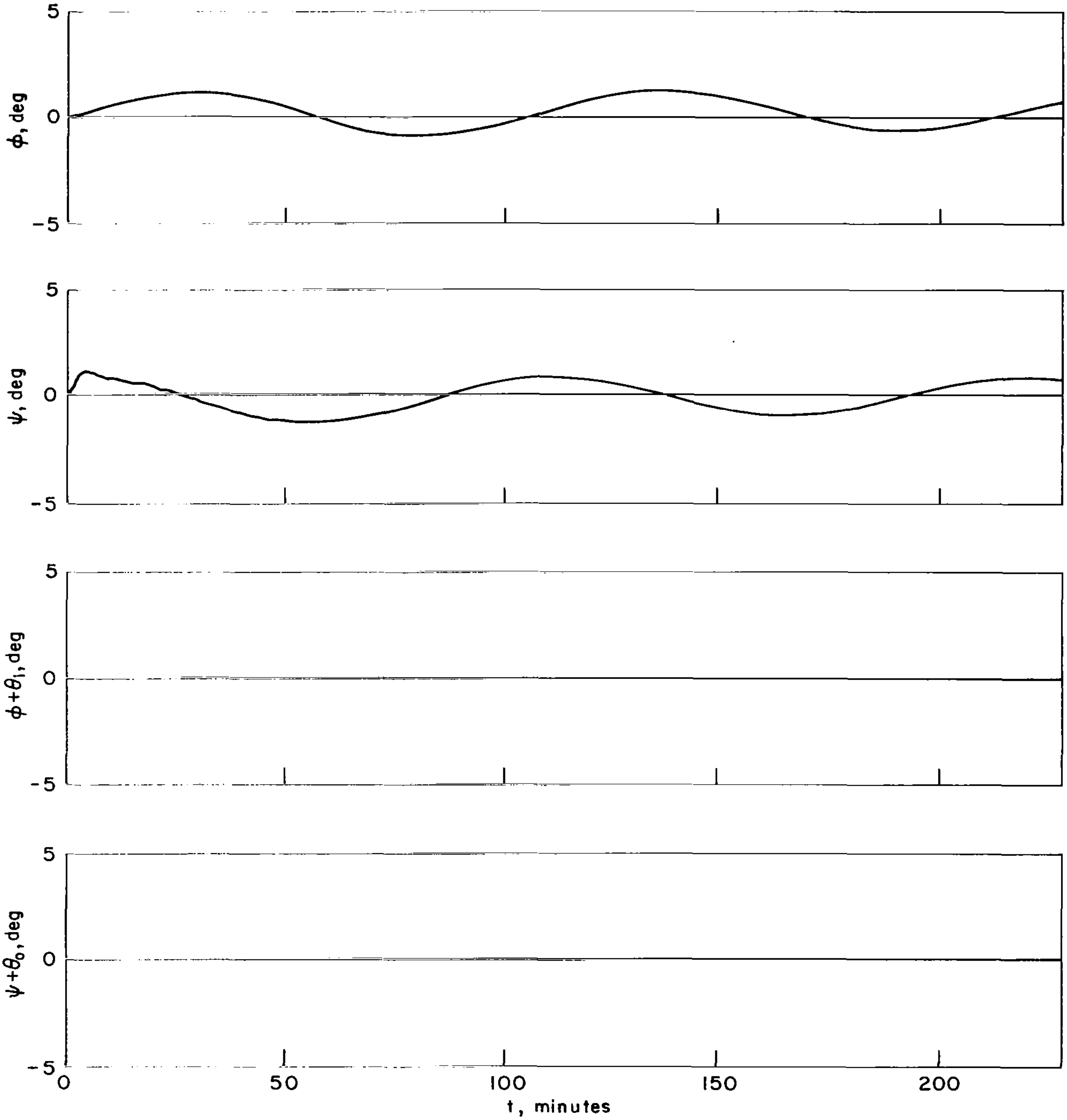
(a) Initial yaw error.

Figure 3.- Time history of attitude. Gyroscope damped about inner gimbal only with light restoring moment command to inner gimbal; $C_i/h = 0.5$, $K_2/h = 0.05$, $C_o/h = 0$.



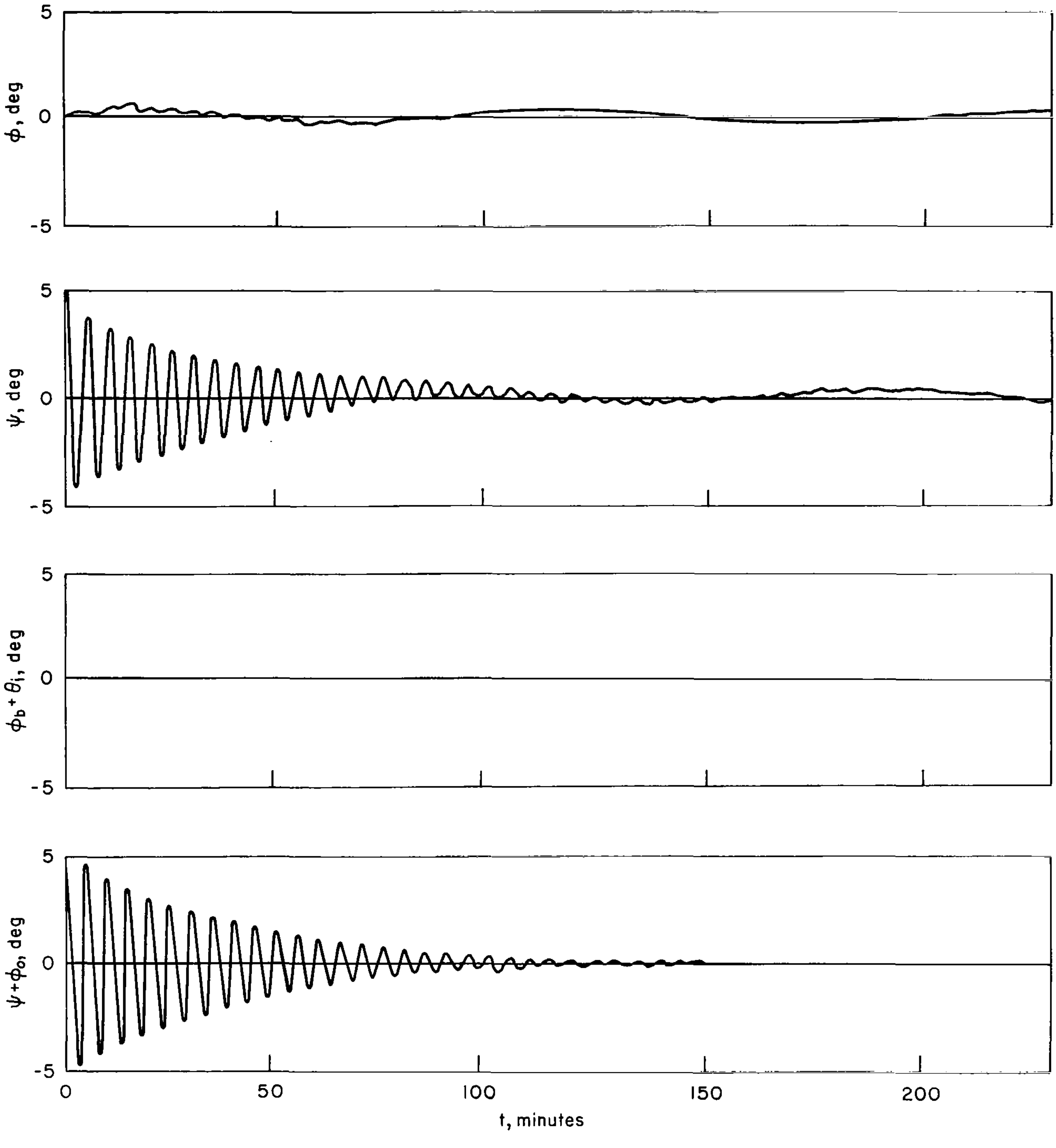
(b) Initial roll rate error.

Figure 3.- Continued.



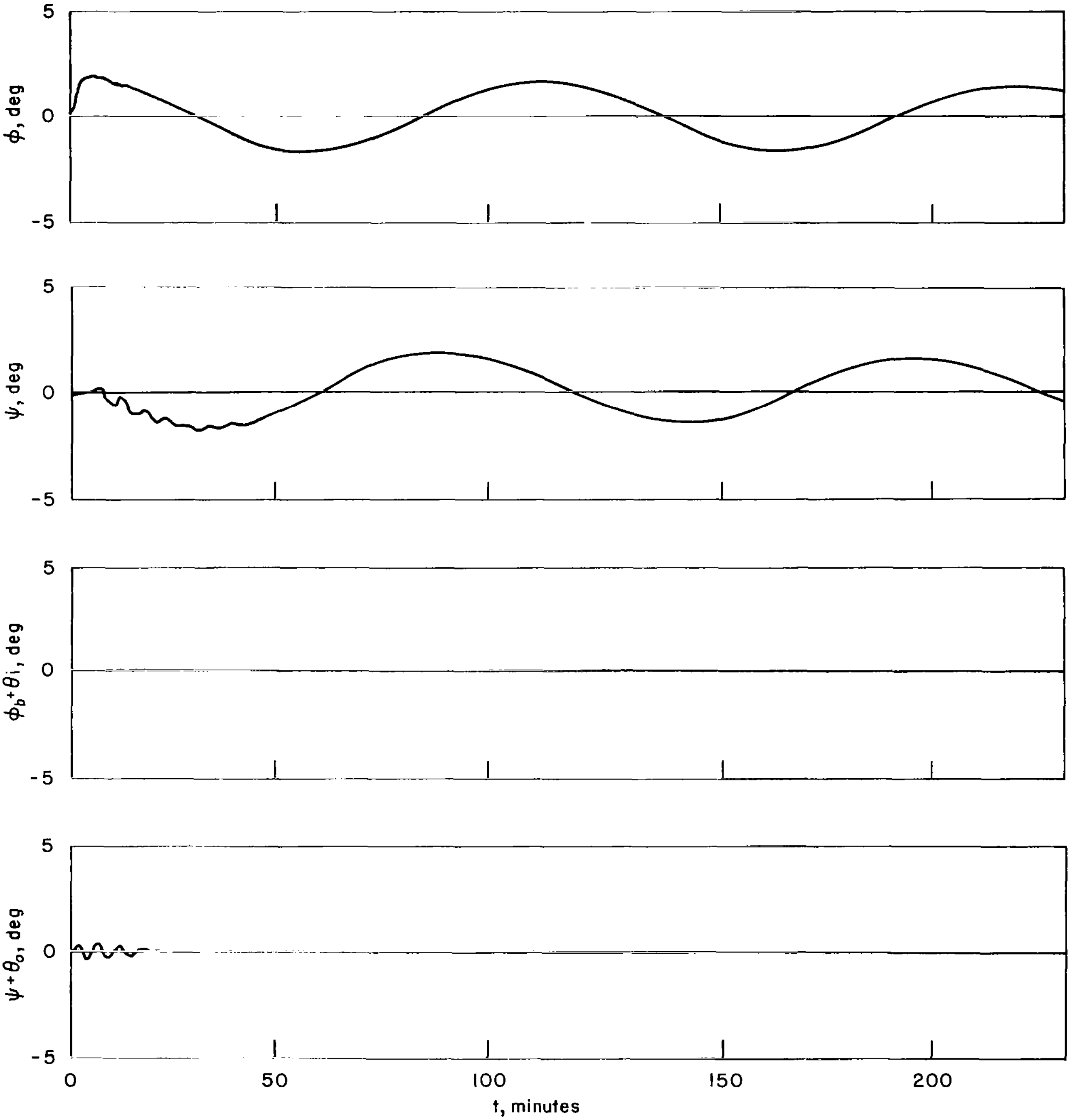
(c) Initial yaw rate error.

Figure 3.- Concluded.



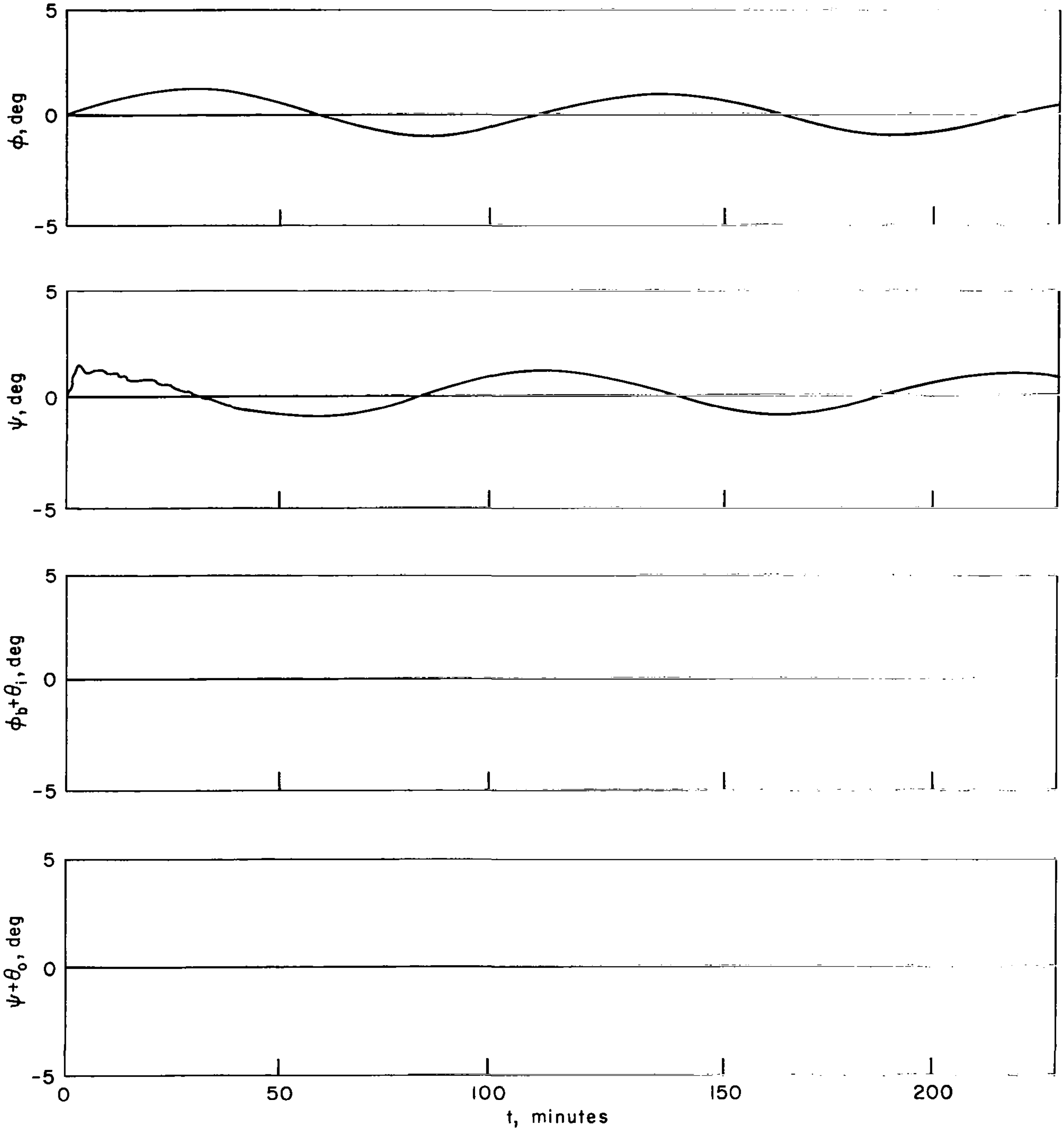
(a) Initial yaw attitude error.

Figure 4.- Time history of attitude. Gyroscope damped about inner gimbal only with high restoring moment command to inner gimbal; $C_i/h = 0.5$, $K_2/h = 0.5$, $C_o/h = 0$.



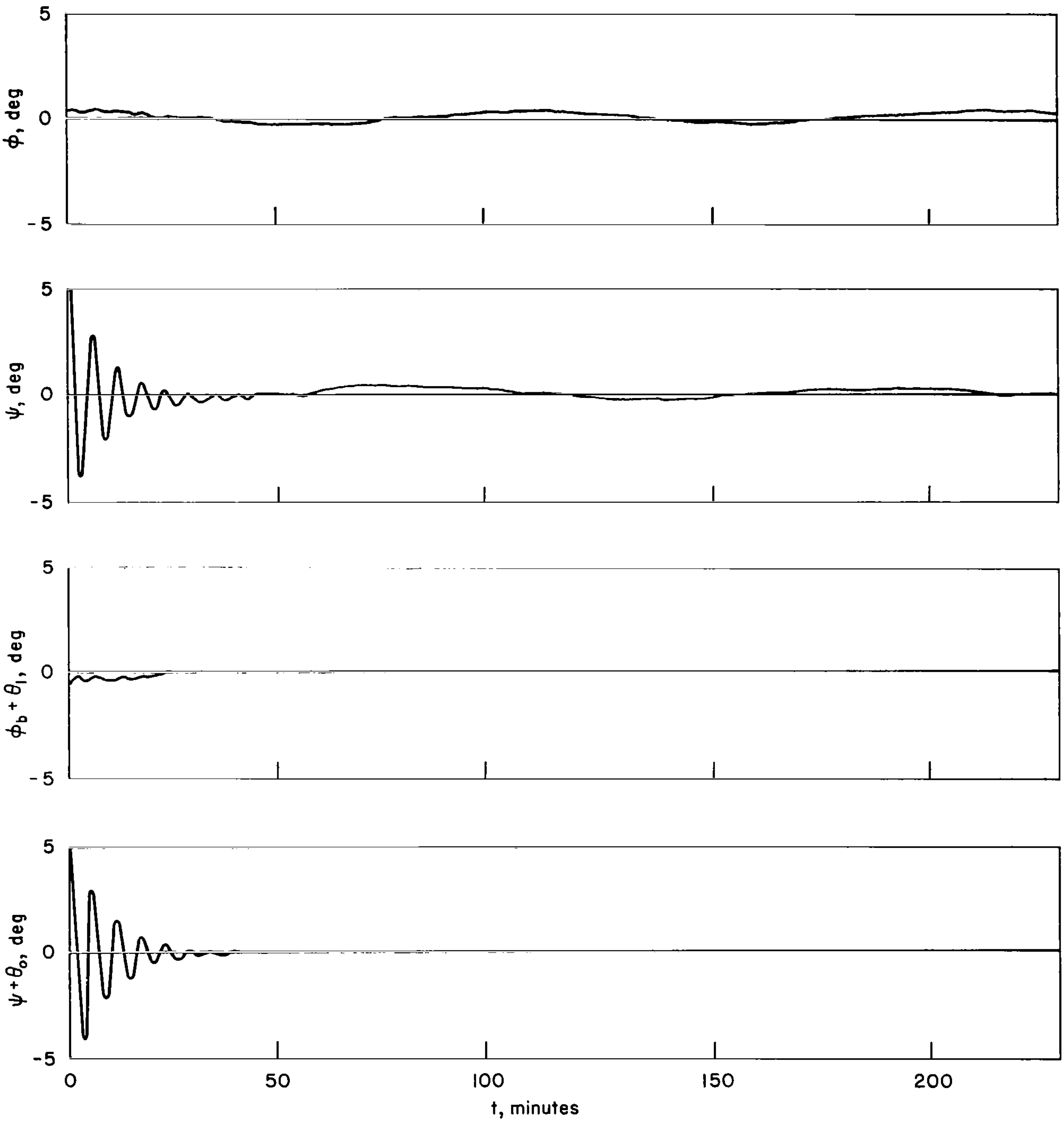
(b) Initial roll rate error.

Figure 4.- Continued.



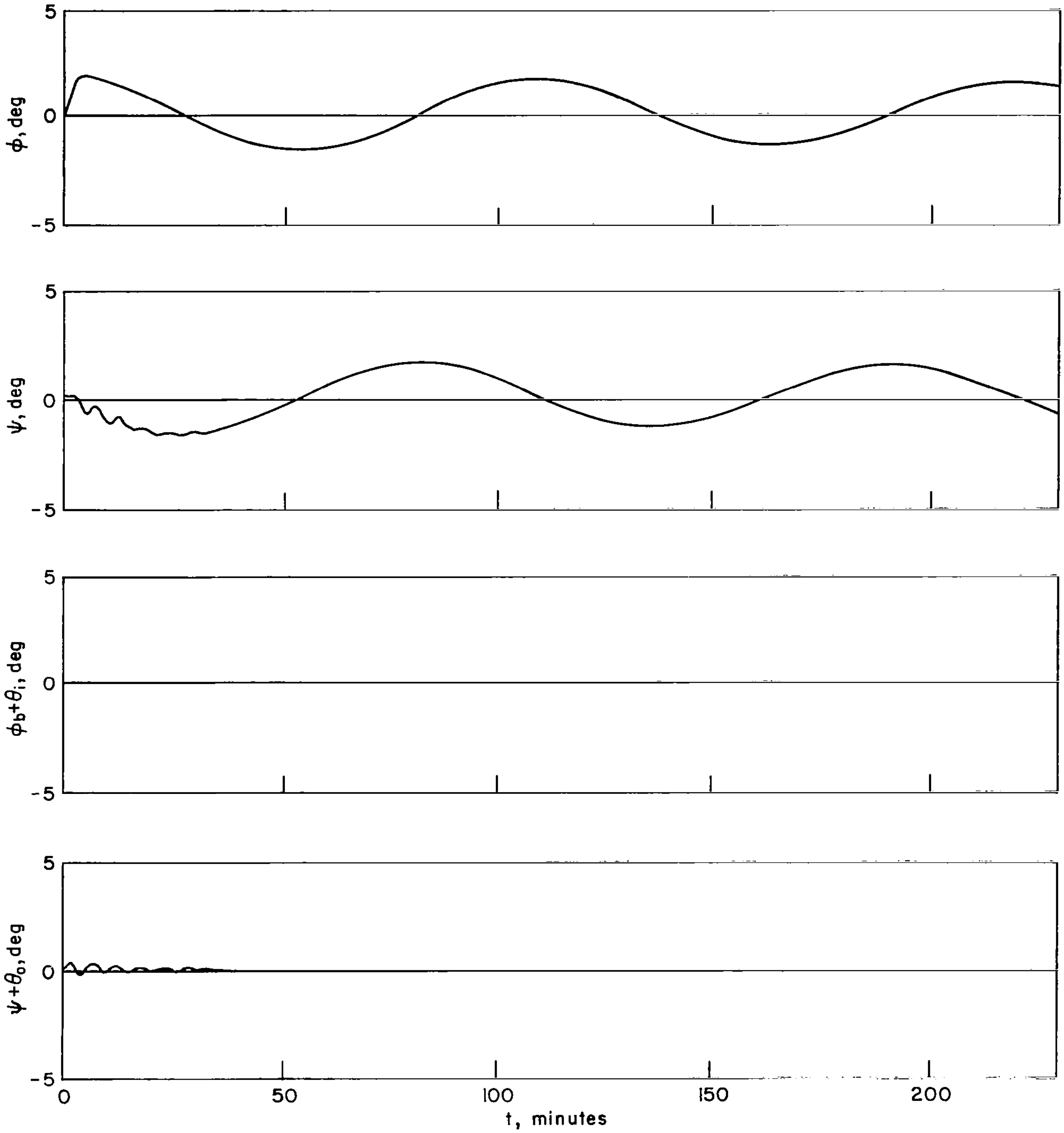
(c) Initial yaw rate error.

Figure 4.- Concluded.



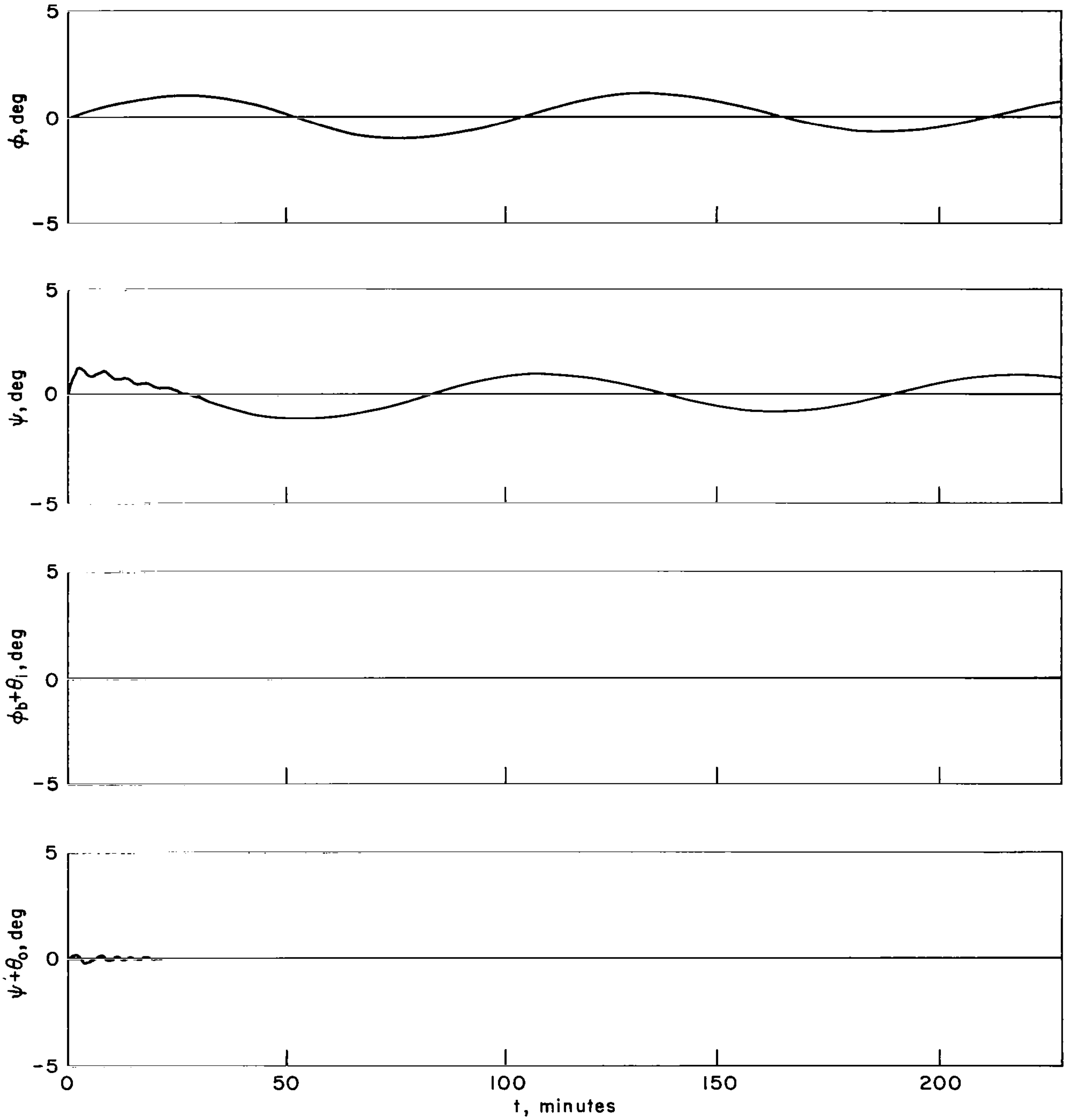
(a) Initial yaw attitude error.

Figure 5.- Time history of attitude. Gyroscope damped about inner and outer gimbal with a high restoring moment command to inner gimbal; $C_i/h = 0.5$, $C_o/h = 0.01$, $K_2 = 0.5$.



(b) Initial roll rate error.

Figure 5.- Continued.



(c) Initial yaw rate error.

Figure 5.- Concluded.

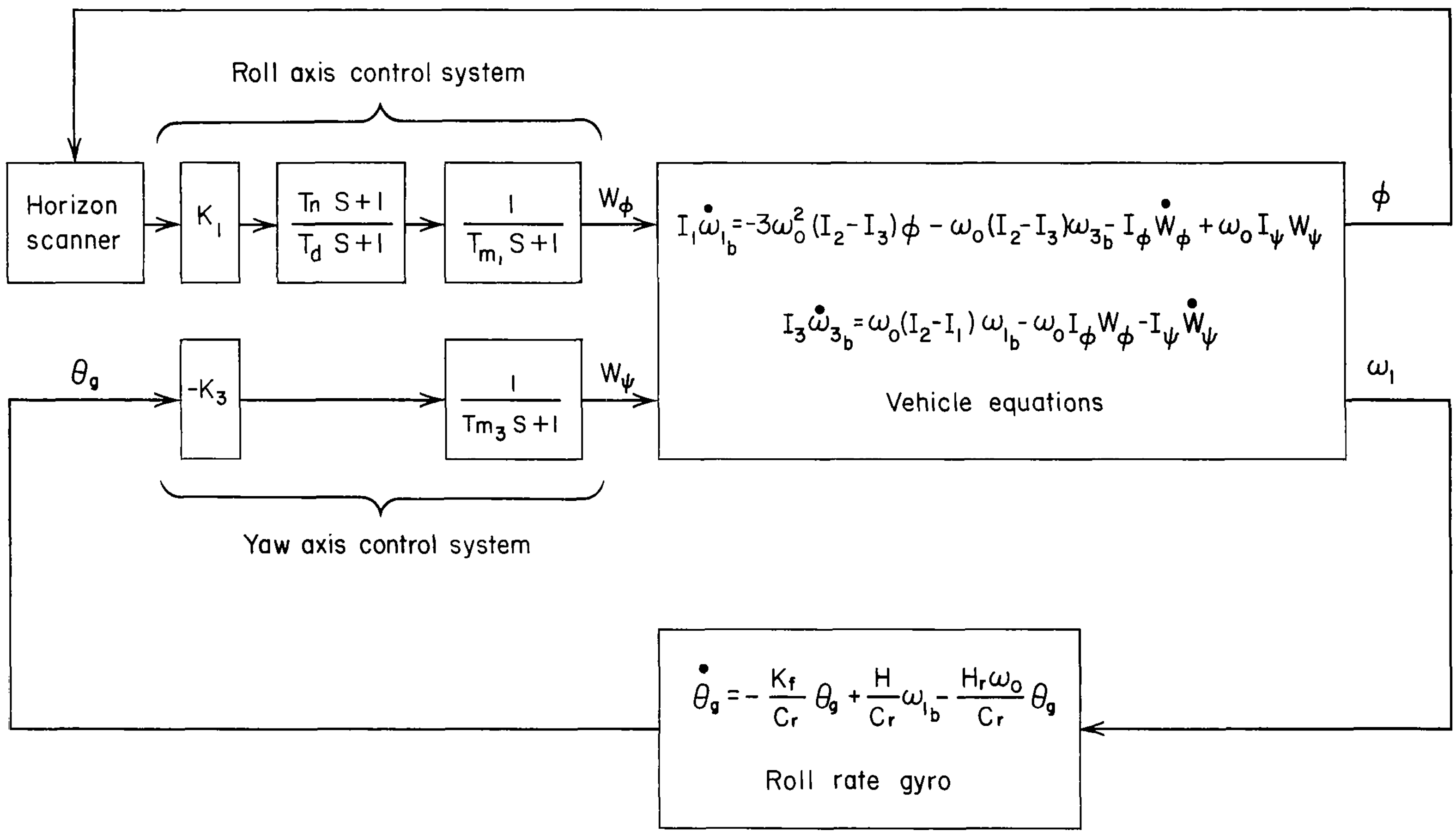
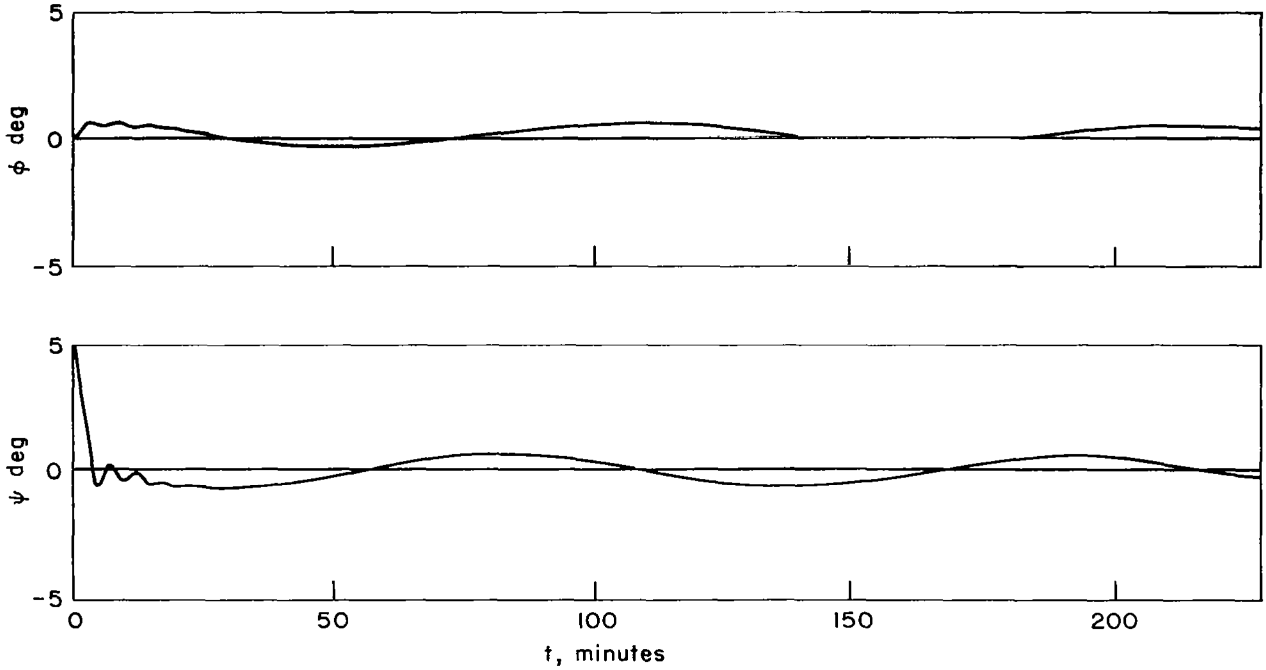
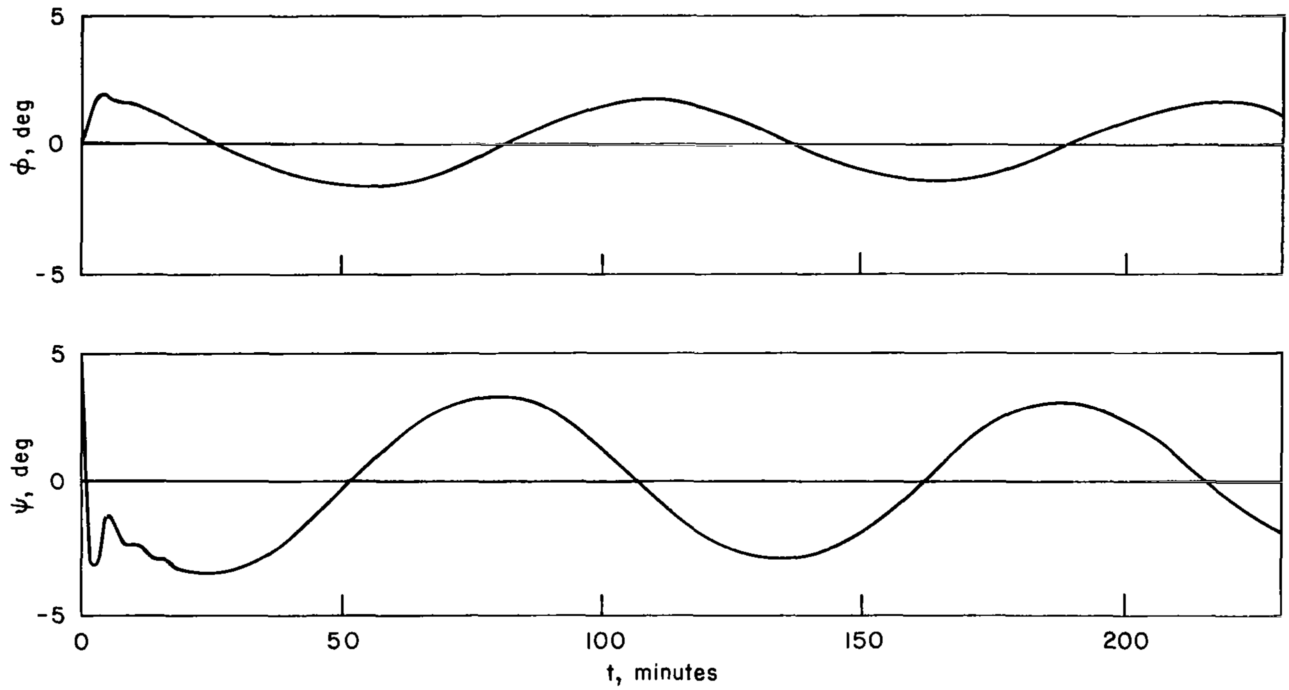


Figure 6.- Block diagram of system and associated control system using a roll rate gyroscope as a yaw attitude sensor.



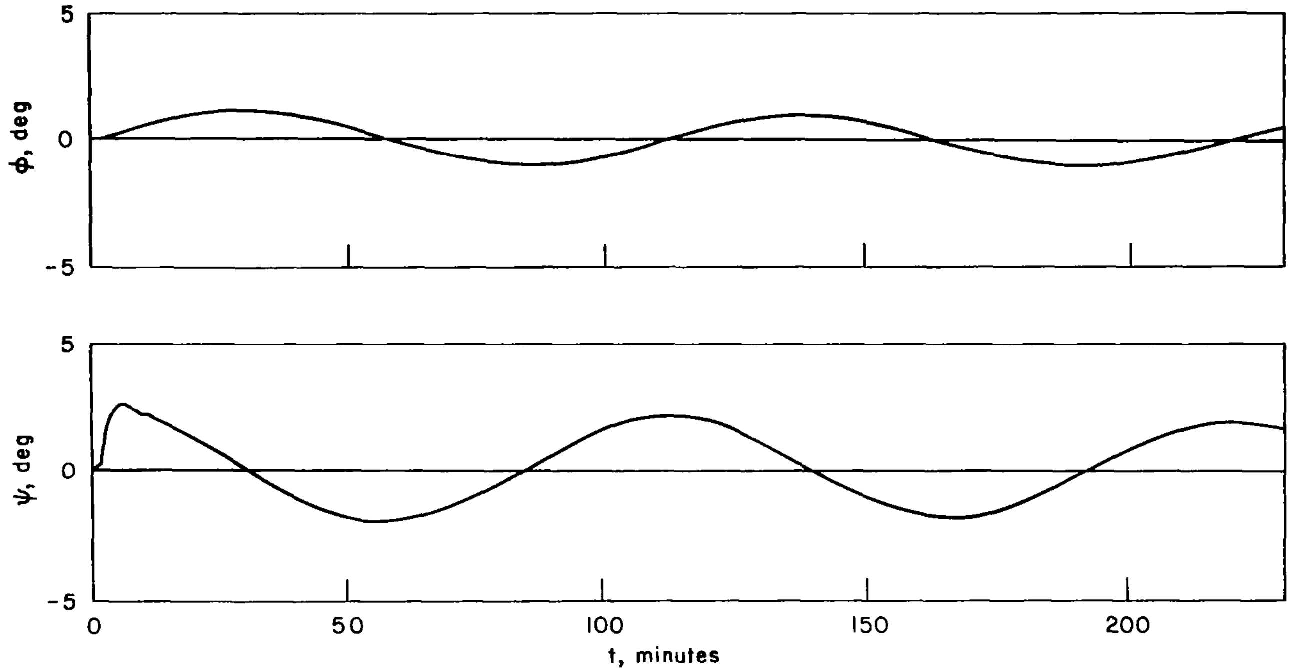
(a) Initial yaw attitude error.

Figure 7.- Time history of vehicle attitude. Roll rate gyroscope system.



(b) Initial roll rate error.

Figure 7.- Continued.



(c) Initial yaw rate error.

Figure 7.- Concluded.